TE equation

The equation used in the Excel spreadsheet ECT*.xls* from www.thetropicalevents.com to calculate the approximate time of a solstice or equinox, *TE*, is as follows:

$$eTE = aTE + \Delta bTE + \Delta mTE + \Delta eTE$$

eTE is an *eccentric* Tropical Event: a solstice or equinox calculated from pure ellipse, unperturbed by the Moon and planets.

aTE is an *average* Tropical Event: calculated from a circular orbit, with an *average* precession rate and an *average* sidereal mean motion. The initial calculations are in dynamical time, so that the quantity ΔT, (dynamical time) – (Universal Time), needs to be included for studies within calendars. Use in a "Copernican" calendar includes an estimation of Universal Time.

$$aTE = Y \times aty$$

aty is the length of the *average* tropical year, calculated from the *average* sidereal year and the *average* precession rate. 1.0000010159 AU is used for the million year average Earth-Sun distance (Quinn, Tremaine, and Duncan 1991), so that an approximate value ~365.256362 is used for the average sidereal year. 50.4712 arcseconds per Julian year is used for the average rate of precession (Laskar 1993), so that a value ~365.242138 is used for *aty*. A good example is 365 $^{77}/_{318}$, so that

$$aty = \frac{116147}{318}$$

Y is the number of average tropical years from the initial epoch Y_0. It rather simplifies the inclusion of an estimated ΔT to set Y_0 close to the epoch in which dynamical time = estimated Universal Time, and further simplifies calculations of solstices and equinoxes to set it as one of the average Tropical Events. We use the average Winter Solstice (northern hemisphere) *aWS* of 1819 as the initial epoch, as it is the closest Tropical Event to the *minimum* of the "best fit" parabola from Morrison and Stephenson 2004 for a long-term estimated $\Delta T = -20 + 32t^2$ seconds, where t = (*year* – 1820)/100, so that

$$Y = year + \frac{Q}{4} - 1820$$

where *year* is the Gregorian calendar year. Q is the "quarter" year, where $Q = 1$ represents an average Vernal Equinox *aVE*; $Q = 2$ an average Summer Solstice *aSS*; $Q = 3$ an average Autumnal Equinox *aAE*; $Q = 4$ an average Winter Solstice *aWS*

As an example we will calculate an approximate time for the Vernal Equinox of 2010 in the Gregorian calendar

$Y = 2010 + 1/4 - 1820 = 190.25$

$aTE = 190.25 \times 116147 / 318 = d\ 69487.3168239$

d is the *day number* in dynamical time, counted from Y_0 as 86400 seconds SI per *day*. It is the same time scale used in the familiar *Julian day numbers*, but shifted by the amount Δ*syst*, about 20 seconds SI, as the Julian day numbers are set so that dynamical time = Universal Time at approximately the year 1900, and we are using the approximate year 1820. It further simplifies a calendar construction to set the initial epoch Y_0 at the beginning of a civil day in Greenwich, so that

JDE of Y_0 = 2385782.5 − Δ*syst* = *JDE* 2385782.499769

where Δ*syst* = 0.000231 *days*, exactly 19.9584 seconds (20/86400 cannot be exactly represented as a decimal).

JDE of *aTE* = *aTE* + (*JDE* of Y_0) = 69487.3168239 + 2385782.499769 = *JDE* 2455269.8165929

This is March 14, 2010 at about 7:30 AM in Greenwich, 6½ days sooner than the true Vernal Equinox, but provides a good "first approximation" from which to calculate the three delta's of the *TE* equation. Even for the 30 million years of Laskar's La93(0,1) data this first approximation, the *aTE*, differs from the *eTE* by no more than ±20 *days* of dynamical time *TD*. The *eTE* can differ from a *true* solstice or equinox by ±20 *minutes TD*: more than adequate for long term calendar studies.

ΔbTE

ΔbTE is the least significant of the 3 delta's, but is the most elusive, as it requires integral knowledge of the sidereal mean motion. The only data readily available is from Laskar (1986) given in terms of t, units of 10000 Julian years from J2000, for the (heliocentric) mean longitude of Earth × 10^{10}, in radians, from the fixed Vernal Equinox of J2000:

$$17534703144 + 628307584918000t - 9793168t^2 + 429738t^3 + 734935t^4 + 83525t^5 - 59447t^6 - 52555t^7 + 13798t^8 + 14426t^9 - 564t^{10} \quad (1)$$

and is valid for ±10000 Julian years from J2000

The derivative of this formula reveals the sidereal mean motion

For epochs outside the range −10000 to +10000 the sidereal year can simply be the same as the average sidereal year *asy*, and if one is using 365 $^{77}/_{318}$ for the average tropical year *aty*, for consistency with the average precession of La93(0,1), 50".4712 per Julian year, one should use a value close to 365.256362539, and 365 $^{413}/_{1611}$ works just fine, so that

$$asy = \frac{588428}{1611}$$

The average speed of the precession during the average sidereal year *asy* is indicated by asp_{asy}

$a_1 = den_{asy} \times num_{aty} = 1611 \times 116147 = 187112817$

$b_1 = a_1 - den_{aty} \times num_{asy} = 187112817 - 318 \times 588428 = 7287$

$gcf_1 = gcf(a_1,b_1) = 3$, the greatest common factor of a_1 and b_1

the constant $a = a_1 / gcf_1 = 187112817 / 3 = 62370939$

the constant $b = b_1 / gcf_1 = 7287 / 3 = 2429$

$asp_{asy} = a / b = 25677.62000823384108...$

asp_{Jy} indicates the average speed of the precession during a Julian year, 365.25 *days*

$asp_{Jy} = asp_{asy} \times asy / 365.25 = \frac{a \times 588428 \times 4}{b \times 1611 \times 1461} = \frac{146803227575568}{5717066859} = \frac{273376587664}{10646307} = 25678.0673020231...$, slightly large, showing 50".47108 per Julian year, but quite good enough for approximate times of solstices and equinoxes for the 30 million years of Laskar's data. If one used *asy* = 365 $^{5097}/_{19882}$ it would show 50".471199, closer to the recommended value of 50".4712

The constant $\Delta bTE_{J2000} = -0.13$ *days*

$\sim aTE = JDE$ of $aTE - 2451545 = 2455269.8165929 - 2451545 = 3724.8165929$

(avg trop yrs from J2000) = $\sim aTE / aty = 3724.8165929 \times 318 / 116147 = 10.1982115469379$

T_{aTE} is the time T in Julian millennia from J2000, at the time of the *aTE*

$T_{aTE} = \sim aTE / 365250 = 3724.8165929 / 365250 = 0.010197992040794$

Using Laskar 1986 formula (1)

$$\text{(radians from J2000)} = \frac{628307584918000t - 9793168t^2 + 429738t^3 + 734935t^4 + 83525t^5 - 59447t^6 - 52555t^7 + 13798t^8 + 14426t^9 - 564t^{10}}{10^{10}}$$

where $t = T_{aTE}/10 = 0.0010197992040794$

(radians from J2000) = 64.0747575006

(sid yrs from J2000) = (radians from J2000) / 2π = 64.0747575006 / 2π = 10.197814383635

asy(T) is the average length of the sidereal year during time span T, where T is Julian millennia from J2000

asy(T) = ~*aTE* / (sid yrs from J2000) = 3724.8165929 / 10.197814383635 = 365.256363057307

A_1 = a × num_{asy} = 62370939 × 588428 = 36700806893892

B_1 = b × den_{asy} = 2429 × 1611 = 3913119

gcf_2 = gcf(A_1 , B_1) = 537, the greatest common factor of A_1 and B_1

the constant A = A_1 / gcf_2 = 36700806893892 / 537 = 68344146916

the constant B = B_1 / gcf_2 = 3913119 / 537 = 7287

avg bty(T) = asy(T) / (1 + B × asy(T) / A)

avg bty(T) = 365.256363057307 / (1+7287 × 365.256363057307 / 68344146916) = 365.242138914285

~*bTE* = (avg bty(T)) × (avg trop yrs from J2000) = 365.242138914285 × 10.1982115469379 = 3724.81659850399

~Δ*bTE* = ~*bTE* − ~*aTE* = 3724.81659850399 − 3724.8165929 = 0.00000560399

Δ*bTE* = ~Δ*bTE* + ΔbTE_{J2000} = 0.00000560399 – 0.13 = −0.12999439601 *days*

Outside the range −10000 to +10000 one can simply set Δ*bTE* = 0, and then add ΔbTE_{J2000} to ΔmTE_{Y0} in the following section. The additional error in calculating a solstice or equinox *without* including Δ*bTE* from −10000 to +10000 can be ±1½ hours *TD*. For an interesting study of *possible* long term effects due to the variations of amplitudes (*possibly* ±5 hours) use (with caution) the "fictitious rendition" for the length of the sidereal year from ECT*.xls*

Δ*mTE*

Δ*mTE* is a value calculated from the precession pA. It can be calculated by the following method for the entire range of La93(0,1) data, with excellent results by the use of Lagrange interpolation polynomials (see Meeus 1998, or documentation for the "Orbital Elements" Excel add-in at thetropicalevents.com). For this example we use formulae from Laskar 1986

(pA from J2000), valid for ±10000 Julian years from J2000:

$$(502909.66t + 11119.71t^2 + 77.32t^3 - 2353.16t^4 - 180.55t^5 + 174.51t^6 + 130.95t^7 + 24.24t^8 - 47.59t^9 - 8.66t^{10}) \times \pi/(180 \times 60 \times 60) \qquad (2)$$

where $t = T_{aTE}/10$ = 0.0010197992040794

(pA from J2000) = 0.00248650482258583 radians

The following if statement converts various data between −3π and +3π to a value between –π and +π

(pA ±pi) = IF((pA from J2000)>π, (pA from J2000) − 2π,IF((pA from J2000)<= −π, (pA from J2000) + 2π, (pA from J2000)))
= 0.00248650482258583

(pA/2pi circle) = IF((pA ±pi)<0, (pA ±pi)/2π +1, (pA ±pi)/2π) = 0.000395739533536371

(total avg cycles) = ~*aTE* / (365.25 × asp_{Jy}) = ~*aTE* × b / (*asy* × a) = 3724.8165929 × 2429 × 1611/ (62370939 × 588428)
= 0.000397147959807337

(~cycles±1 from J2000) = INT(total avg cycles) + (pA/2pi circle) = 0.000395739533536371
where INT(*x*) is the integer less than or equal to *x*

(T_{aTE} at Y_0) = ((*JDE of* Y_0) − 2451545) / 365250 = −0.180047913021219

(pA at Y_0): using Laskar 1986 precession, formula (2), with t = (T_{aTE} at Y_0)/10 = −0.0180047913021219

(pA at Y_0) = −0.0438813563532027 radians

(cycles Y_0 to aTE) = (~cycles±1 from J2000) − ROUND((~cycles±1 from J2000) – (total avg cycles) ,0) – (pA at Y_0) / 2π
= 0.000395739533536371 − 0 + 0.0438813563532027 / 2π = 0.007379674307
where ROUND(x,0) rounds to the integer closest in value to x

(Msidereal orbits) = Y × *aty* / *asy* = Y × a / (a + b) = 190.25 × 62370939 / (62370939 + 2429) = 190.242591112765

(total mean tropical years from Y_0 to aTE) = (Msidereal orbits) + (cycles Y_0 to aTE)
= 190.242591112765 + 0.007379674307 = 190.249970787072

~*mTE* = Y × *aTE* / (total mean tropical years from Y_0 to aTE) = 190.25 × 69487.3168239 / 190.249970787072
= 69487.3274936939

The constant ΔmTE_{Y0} = 8.422 *days*, given to the nearest $^1/_{1000}$ of a day, indicating that the intended accuracy at this point in the process is ±0.0005 *days*, or about 1 *minute TD*.

ΔmTE = ~*mTE* – *aTE* + ΔmTE_{Y0} = 69487.3274936939 − 69487.3168239 + 8.422 = 8.4326697939 *days*

ΔmTE varies by about ±12 *days* during the entire 30 million years of Laskar's La93(0,1) data

$$mTE = aTE + \Delta bTE + \Delta mTE$$

mTE = *aTE* + ΔbTE + ΔmTE = 69487.3168239 − 0.12999439601 + 8.4326697939 = *d* 69495.6194992979

JDE of *mTE* = *mTE* + (*JDE* of Y_0)

JDE of *mTE* = 69495.6194992979 + 2385782.499769 = *JDE* 2455278.1192683

This is March 22, 2010 at about 3 PM in Greenwich, almost 2 days later than the true Vernal Equinox.
The *mean* Tropical Event *mTE* represents the mean planet (as if we were in circular orbit) from the moving Vernal Equinox (including the precession), so that the longitude of the mean planet at the time T_{mTE} ,

T_{mTE} = (*JDE* of *mTE* – 2451545) / 365250 = (2455278.1192683 – 2451545) / 365250 = 0.0102207235271732,

is a 90° increment (a solstice or equinox), with an intended accuracy of ±0.00001 radians (about 1 minute of time). It should be noted that the constant ΔmTE_{Y0} is intended to include the "constant of aberration" for the "observed" Sun, about 0.000099 radians. Using T_{mTE} /10 in equations (1) and (2) results in 65.9710514681048 + 0.00249204740661573 – 0.000099
= 65.9734445155114 , which reduces to 3.14159144371554 radians, 0.00000120987425 less than π (heliocentric longitude of the Vernal Equinox), well within our intended accuracy. Note: alternately one could skip the previous two sections by instead using an iterative scheme to find the time T_{mTE} of the mean planet at the required 90° increment, and then proceed as follows.

Δ*eTE*

ΔeTE is a generic term for each of the 4 values associated with each of the 4 Tropical Events, and is calculated from the eccentricity e and perihelion from the moving equinox ϖ, including the indication of the speed of the precession *sp* and the sidereal year *sy*, so that we can use the mean tropical year *mty*. The following method also applies to the use of Laskar's data for k and h to determine e and ω, and the derivatives of precession, again best by using Lagrange interpolation polynomials.

dpA: using the derivative of the precession in radians (Laskar 1986) per 1000 Julian years

$$dpA = (502909.66 + 22239.42t + 231.96t^2 - 9412.64t^3 - 902.75t^4 + 1047.06t^5 + 916.65t^6 + 193.92t^7 - 428.31t^8 - 86.6t^9) \times \pi/(10 \times 180 \times 60 \times 60)$$

with $t = T_{\mathrm{mTE}}/10$ = 0.00102207235271732

dpA = 0.243828503601382 radians per 1000 Julian years

Using the derivative of mean longitude of Earth from Laskar 1986 to determine the length of the sidereal year *sy*

$$sy = \frac{2\pi \times 3652500 \times 10^{10}}{628307584918000 - 19586336t + 1289214t^2 + 2939740t^3 + 417625t^4 - 356682t^5 - 367885t^6 + 110384t^7 + 129384t^8 - 5640t^9}$$

with $t = T_{mTE}/10 = 0.00102207235271732$

$sy = 365.256363062974$

$sp_{sy} = 2\pi \times 365250/(dpA \times sy) = 2\pi \times 365250/(0.243828503601382 \times 365.256363062974) = 25768.4222975507$

$mty = sy \times sp_{sy} / (sp_{sy}+1) = 365.256363062974 \times 25768.4222975507/25769.4222975507 = 365.2421890408$

Next we need the eccentricity e and perihelion in radians from the fixed equinox of J2000 ω, calculated from k and h
Using Laskar 1986, again valid for ±10000 Julian years from J2000:

$$k = \frac{-37408165 - 82266699t + 27626329t^2 + 11695572t^3 - 2695722t^4 - 715070t^5 + 218146t^6 + 22635t^7 - 19921t^8 - 2032t^9 + 475t^{10}}{10^{10}}$$

$$h = \frac{162844766 - 62030259t - 33829810t^2 + 8510121t^3 + 2770542t^4 - 467407t^5 - 62395t^6 + 247t^7 + 403t^8 + 686t^9 - 423t^{10}}{10^{10}}$$

with $t = T_{mTE}/10 = 0.00102207235271732$

$k = -0.00374922186467685$

$h = 0.0162781331256635$

e = SQRT(k^2+h^2) = 0.0167043192811738 , where SQRT(x) is $\sqrt{x}$

ω = ATAN2(k,h) = 1.79717108039395 , where ATAN2(k,h) is the inverse tangent of h/k, between $-\pi$ and $+\pi$ radians

$\varpi_1 = \omega$ + (pA ±pi) = 1.79717108039395 + 0.00248650482258583 = 1.79965758521654

note: here it is more accurate to recalculate (pA ±pi) using T_{mTE} , but the difference is minimal compared to the accuracy of this section: ±20 minutes *TD*, due to the perturbations of the Moon (±10 mins) and of the planets (±10 mins).

converted to an angle between 0 and 2π for perihelion from the moving equinox ϖ
ϖ = IF(ϖ_1>=2π, ϖ_1-2π,IF(ϖ_1<0, ϖ_1+2π, ϖ_1)) = 1.79965758521654

Now we use the "equation of center" *C*. From Meeus 1998 p.275 and the "Equation of Kepler" we find the true anomaly *v*, the eccentric anomaly *E*, and the mean anomaly *M*

$$v \text{ at } VE = \pi - \varpi$$

$$v \text{ at } SS = 3\pi/2 - \varpi$$

$$v \text{ at } AE = 2\pi - \varpi$$

$$v \text{ at } WS = \pi/2 - \varpi$$

$$tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}} \quad tan\frac{v}{2}$$

$$M = E - e\ sinE$$

$$-C = M - v$$

$$n_t = \frac{2\pi}{mty}$$

$$\Delta eTE = \frac{-C}{n_t}$$

v at $VE = \pi - \varpi$ =3.14159265358979-1.79965758521654 = 1.34193506837325

E =ATAN(SQRT((1-e)/(1+e)) × TAN(v/2)) × 2 = 1.32569630256058

M =E-e $sinE$ =1.32569630256058-0.0167043192811738 × SIN(1.32569630256058) = 1.30949122429074

-C = $M - v$ =1.30949122429074-1.34193506837325 = -0.0324438440825101

n_t = 2π / mty =6.28318530717959/365.2421890408 = 0.0172027917248019

ΔeTE = -C/n_t =-0.0324438440825101/0.0172027917248019 = -1.885963895 *days*

Alternately, by calculating the lengths of the seasons, one can use the "Ancient Astronomer's Formula" to calculate ΔeTE

$$\Delta eVE = \frac{F + 3W - 3V - S}{8}$$

$$\Delta eSS = \frac{W + 3V - 3S - F}{8}$$

$$\Delta eAE = \frac{V + 3S - 3F - W}{8}$$

$$\Delta eWS = \frac{S + 3F - 3W - V}{8}$$

E_{WS} =ATAN(SQRT((1-e)/(1+e)) × TAN((π/2-ϖ)/2)) × 2 = -0.225101894822176

ϖ to WS =(E_{WS} -e × SIN(E_{WS})) × mty/2π = -12.8684575907845

E_{VE} =ATAN(SQRT((1-e)/(1+e)) × TAN((π-ϖ)/2)) × 2 = 1.32569630256058

ϖ to VE =(E_{VE} -e × SIN(E_{VE})) × mty/2π = 76.1208555703667

E_{SS} =ATAN(SQRT((1-e)/(1+e)) × TAN((3π/2-ϖ)/2)) × 2 = 2.90891036561448

ϖ to SS =(E_{SS} -e × SIN(E_{SS})) × mty/2π = 168.87134312852

E_{AE} =ATAN(SQRT((1-e)/(1+e)) × TAN((2π-ϖ)/2)) × 2 = -1.78335716116808

ϖ to AE =(E_{AE} -e × SIN(E_{AE})) × mty/2π = -102.717560011001

W_1 =(ϖ to VE)-(ϖ to WS) = 88.9893131611513

V_1 =(ϖ to SS)-(ϖ to VE) = 92.7504875581535

S_1 =(ϖ to AE)-(ϖ to SS) = -271.588903139521

F_1 = (ϖ to WS)-(ϖ to AE) = 89.8491024202163

W =IF(W_1<0, W_1+ mty, W_1) = 88.9893131610537 *days* of Winter

V =IF(V_1<0, V_1+ mty, V_1) = 92.7504875581535 *days* of Spring

S =IF(S_1<0, S_1+ mty, S_1) = -271.588903139521 + 365.2421890408 = 93.653285901279 *days* of Summer

F =IF(F_1<0, F_1+ mty, F_1) = 89.8491024202163 *days* of Fall

ΔeVE = (F+3×W-3×V-S)/8 = -1.885963334 *days*

The two methods calculate differently by 0.00000056 *days* (0.048 *seconds*), and one assumes that this is due to “decimal rounding”, and the “double precision” used by Excel, and most computer software.

$$eTE = mTE + \Delta eTE$$

$eVE = mTE + \Delta eVE$ = 69495.6194992979 -1.885963334 = d 69493.7335359639

JDE of $eVE = eVE + (JDE$ of $Y_0)$ = 69493.7335359639 + 2385782.499769 = JDE 2455276.23330496

ΔT

ΔT, the difference (dynamical time) – (Universal Time), is by far the most elusive quantity for long term calendar studies. However, using the study of Morrison and Stephenson (2004), Espenak and Meeus have developed a great set of equations for the years −1999 to +3000 available at http://eclipse.gsfc.nasa.gov/SEcat5/deltatpoly.html . We prefer to use them only for the years −404.15 to +2003.45, as the following “sum of 4 sines” more accurately follows the Morrison and Stephenson (2004) study from −500 to −2000, and allows an extension even further. This equation for ΔT in seconds is

```
-3.0169675+0.003390245877*Y^2
+(11.85034251*Sin(4.521017826+0.00009728265802*Y)/0.00009728265802+119589.730883314)
+(4.889524586*Sin(1.038218036+0.0003290065396*Y)/0.0003290065396-12803.1780446892)
+(1.416055354*Sin(3.116643354+0.004835133099*Y)/0.004835133099-7.30609135253061)
+(1.209213516*Sin(0.4888837632+0.004072065294*Y)/0.004072065294-139.46138984887)
```

where Y can be d/aty , and is valid from Y = −7727.87259149758 to Y = 1165.47962600512 (5908 BC to 2985 AD)

To facilitate use of this formula unassociated with the above “method”, calculate Y as (JDE − 2385782.5) / 365.2421378
It should be noted by the use of "*JDE*" that this is intended to convert dynamical time into Universal time.

This single equation is no more than 15 seconds in error for the entire time span for the recorded values of ΔT, ~1600 to 2010. One must include the errors associated with these recorded values. For example, the year 1600 is often given as 120 seconds, with an associated error of ±20 seconds. A value of 85 seconds for the year 1600 would therefore be only 15 seconds in error. This equation returns 100.198 secs for the year 1600, so for that particular year it can be considered as having no error.

For the Vernal Equinox of 2010
Y = d / aty = 69493.7335359639 × 318 / 116147 = 190.267568378318 , and the “sum of 4 sines” returns
ΔT = 66.1405146650131 seconds

However, it is often more satisfying to use the *extreme accuracy* of the Espenak and Meeus polynomials, where they use t = y - 2000 , and y is given as: year + (month - 0.5)/12. By this method y calculates the portion of the year at “mid-month” only. To facilitate smooth transitions we prefer to use: y = 1820 + (d -18)/365.2425. When not using the above “method” we recommend calculating y as (JDE – 2451544.5) / 365.2425 + 2000 to obtain smooth transitions for their polynomials.

In order to transition from the Espenak and Meeus polynomials at 2003.45 to the “sum of sines” at 2050 we use a polynomial that also follows the data from the *International Earth Rotation and Reference Systems Service* (IERS) from 2000 thru 2010:

$$63.9 + 0.164954 \times t - 0.00281933 \times t^2 + 0.000879724 \times t^3 - 0.0000104809 \times t^4 \quad \text{where } t = (JDE - 2451544.5) / 365.2425$$

t = (2455276.23330496 − 2451544.5)/365.2425 = 10.2171387638629 , so that the polynomial returns

ΔT = 66.1151192185045 seconds , and dividing by 86400: ΔT_{JDsyst} = 0.00076522 days

JD of eVE = JDE of $eVE - \Delta T_{JDsyst}$ = 2455276.23330496 − 0.00076522 = JD 2455276.23253974

$\Delta T_{Dsyst} = \Delta T_{JDsyst} + \Delta syst$ = 0.00076522 + 0.000231 = 0.00099622 days

D *of* $eVE = eVE - \Delta T_{Dsyst}$ = 69493.73353596 − 0.00099622 = D 69493.73253974

This is March 20, 2010 at 5:35 PM in Greenwich, 4 minutes after the true Vernal Equinox at 5:31 PM (calculated from Bretagnon and Simon 1986, including their formulae for nutation and aberration). This does *not* include the concept of “daylight savings time”.

The following formulae return ΔT in days from "Scenario 10a" in ECT1.6.xlsm, readjusted for standard double precision on an Excel spreadsheet, and Y is (*JDE*-2385782.5)/365.2421378.

From Y = -18080.8569219084 to Y = -7727.87259149758 (16261 BC to 5908 BC):
(-152.934868+0.003390245877*Y^2
+(11.85034251*SIN(4.521017826+0.00009728265802*Y)/0.00009728265802+119589.730883314)
+(4.889524586*SIN(1.038218036+0.0003290065396*Y)/0.0003290065396-12803.1780446892))/86400
+0.000002930134880633*Y+0.0231025306199516

From Y = -1938856065.12609 to Y = -18080.8569219084:
-(1808.41540551846*Y-17802587981659.3*((1+(Y+18080.8569219084)/13463985755.3156)^(35/26)-1)
-69715684935.6374*((1+(Y+18080.8569219084)/13463985755.3156)^(11/2)-1)+32697685.4793694)+1.30023618539E-8

From Y = -4500000000 to Y = -1938856065.12609:
-(1808.41540551846*Y-4011240664619.57*((1+(Y+1938856065.12609)/2561143949.12143)^(14/13)-1)
-18115544946.9063*((1+(Y+1938856065.12609)/2561143949.12143)^2-1)+3402226352645.89)-0.000335693

From Y = +1165.47962600512 to Y = +10682.4663136617 (2985 AD to 12502 AD):
(-152.934868+0.003390245877*Y^2
+(11.85034251*SIN(4.521017826+0.00009728265802*Y)/0.00009728265802+119589.730883314)
+(4.889524586*SIN(1.038218036+0.0003290065396*Y)/0.0003290065396-12803.1780446892))/86400
+0.00000583377118476*Y+0.00762862609185111-6.9389E-18

From Y = +10682.4663136617 to Y = +250000000:
-(1808.4159809978*Y-1810574188741.9*((1+Y/1070750140.18301)^(20/19)-1)
-18105741887.419*((1+Y/1070750140.18301)^(32/19)-1)-0.798712707645782)

From Y = +250000000 to Y = +5000000000:
-(1808.4159809978*Y-1672674863864.45*((1+(Y-250000000)/965163563.979899)^(27/26)-1)
-21153783485.5091*((1+(Y-250000000)/965163563.979899)^(3/2)-1)-455212309211.431)-0.00058841706

For an interesting study of different possible long term effects of ΔT use some of the other scenarios in ECT*.xls*

A single equation for ΔT in seconds from Y = -331.298643742993 to Y = 189.864878763783 (AD 1488 to 2009.813):

12.34471667+0.003390245877*Y^2
+(11.85034251*Sin(4.521017826+0.00009728265802*Y)/0.00009728265802+119589.730883314)
+(4.889524586*Sin(1.038218036+0.0003290065396*Y)/0.0003290065396-12803.1780446892)
+(1.416055354*Sin(3.116643354+0.004835133099*Y)/0.004835133099-7.30609135253061)
+(1.209213516*Sin(0.4888837632+0.004072065294*Y)/0.004072065294-139.46138984887)
+(0.348873982*Sin(2.711947318+0.03080982016*Y)/0.03080982016-4.71677204529075)
+(0.2198280214*Sin(0.6000046074+0.01828535077*Y)/0.01828535077-6.7882249137094)
+(0.4205366394*Sin(5.173509492+0.09238356146*Y)/0.09238356146+4.07662865593031)
+(0.181331671*Sin(6.848632475+0.06187173456*Y)/0.06187173456-1.57028600582809)
+(0.3309858275*Sin(2.696813396+0.1010673329*Y)/0.1010673329-1.40905575588025)
+(0.2713170885*Sin(1.758701847+0.1327208509*Y)/0.1327208509-2.00828480358683)
+(0.175825584*Sin(0.7598732321+0.1997702484*Y)/0.1997702484-0.60626574534795)
+(0.1163835709*Sin(3.160230197+0.2762080472*Y)/0.2762080472+7.85269764264254E-3)
+(0.09261707963*Sin(0.6334367883+0.2793572382*Y)/0.2793572382-0.196242431358713)

Better yet use until Y = 188.614438520988 (AD 2008.563), and then use the polynomial until Y = 230.05 (AD 2050)

For impeccable mathematical accuracy in Excel use the Xnumbers add-in, from the author: Leonardo Volpi, or from an extended version supporting updates since the release of Excel 2007 available at thetropicalevents.com

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For questions or comments email to: steve@thetropicalevents.com